One and two photon quantum interference in a Mach-Zehnder interferometer

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ABSTRACT

A re-visitation of the well known free space Mach Zehnder interferometer is here reported. Coexistence between one-photon and two-photons interference from collinear color entangled photon pairs is investigated. This is seen to arise from an arbitrarily small unbalance in the arm transmittance. The tuning of such asymmetry is reflected in dramatic changes in the coincidence detection, revealing beatings between one particle and two particle interference patterns. Our configuration explores new physics of the real Mach Zehnder interferometer especially useful for quantum optics on a chip, where the guiding geometry forces photons to travel in the same spatial mode.

Keywords: Quantum optics, entangled photons, Mach-Zehnder interferometer, nonlinear optics, SPDC

1. INTRODUCTION

The Mach-Zehnder (MZ) interferometer has long been an important and versatile tool in optics, from its origins in experiments with classical light,^{1,2} to its more recent applications in the study of quantum states.^{3–6} A source of entangled photon pairs is the process of Spontaneous Parametric Down Conversion (SPDC) in a nonlinear optical material.^{3,7,8} After such a photon pair is injected into a MZ interferometer, the photodetection coincidence rate at the two output ports can be measured as a function of the time delay between the long and short arms of the interferometer. In most experiments, the MZ interferometer is excited symmetrically from both ports of the input beamsplitter (BS).^{9,10} As the photons are indistinguishable, the Hong-Ou-Mandel (HOM) effect leads to a bunching of photons exiting this first beamsplitter,^{11–13} suppressing single-photon interference effects in the coincidence rate measured at the output ports. The observed interference pattern then only contains twophoton interference fringes; these have no classical analogue but reflect the entanglement in the input state.^{9,10}

In this paper, we show that much richer physics can be explored by asymmetrically exciting the MZ interferometer, such that collinear photon pairs are sent into the *same* input port. By controlling the relative transmittance of the arms, we can tune from a regime where the interference is dominated by two-photon effects to the opposite limit of single-photon physics. In intermediate regimes, there is a complex interplay between both single-photon and two-photon interference.

This paper is organized as follows: in Section 2, we present a theoretical model of the unbalanced, asymmetricallyexcited MZ interferometer, and derive the probability of a coincidence photodetection in terms of an unbalancing parameter between the two MZ arms. In Section 3, we use our theoretical model to fit experimental results.¹⁴ Finally in Section 4, we compare the interference patterns expected considering both lossless and lossy beamsplitters in the Mach-Zehnder interferometer.

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2. THEORETICAL MODEL

We begin by building a theoretical model of the MZ interferometer sketched in Fig.1. The input radiation state $|\Psi_{in}\rangle$ is sent into a single input port of the first beamsplitter (BS1). By using only one input port, we probe a wider range of interference effects than if both input ports are used. After exiting BS1, the light propagates along the MZ arms, guided by mirrors M1 and M2 into the second beamsplitter (BS2) before being collected by photodetectors C and D.

We model the operation of the beamsplitters as the transformation of the input fields into the output fields according to:

$$BS1 \rightarrow \begin{pmatrix} r_1 e^{-i\delta} & t_1 \\ t_1 & r_1 e^{-i\delta} \end{pmatrix}$$

$$BS2 \rightarrow \begin{pmatrix} r_{2c} e^{-i\delta} & t_{2c} \\ t_{2d} & r_{2d} e^{-i\delta} \end{pmatrix}$$
(1)

where all coefficients r_i and t_i are real, and hence r_i^2 and t_i^2 are the reflectivity and transmittivity respectively. Here we have assumed that the reflection and transmission coefficients are independent of frequency. We also have taken the operation of BS1 to be symmetric, while that of BS2 may be different for the light collected by detector C or D, as indicated by the second subscript c, d. As described in Section 3, experimentally we introduce such an asymmetry by tuning the position of detector D with respect to the output of BS2, thus reducing t_{2d} and r_{2d} relative to r_{2c} and t_{2c} . As we shall show, these coefficients control the amplitude of various photon interference effects and so this controlled asymmetry can be used to tune between different regimes.

In the beamsplitter relation (1), we have also explicitly kept the relative phase δ between the reflection and transmission coefficients. For an ideal lossless beamsplitter, the relative phase must be equal to $\pi/2$ as required by energy conservation.¹⁸ However, as derived by Barnett et. al.,¹⁵ this stringent phase condition may be relaxed when a beamsplitter is intrinsically lossy. This will be discussed in further detail in Section 4. Note that we assume for simplicity that the relative phase is identical and symmetric for both beamsplitters; this is reasonable for this experiment, as the controlled asymmetry for *BS2* introduced above affects the amplitude but not the phase of the light collected.



Figure 1. Sketch of the Mach-Zehnder interferometer, with parameters defined in the text.

To study photon interference, we calculate the coincidence rate of photons arriving at the two photodetectors, which operate with a resolving time T_R and efficiency K. With reference to Fig. 1, the probability per unit time of coincidence detections at the output ports C and D of the MZ at times t and $t + \tau$ is given by:¹⁶

$$P(\tau) = K \left\langle \hat{E}_D^-(t) \hat{E}_C^-(t+\tau) \hat{E}_C^+(t+\tau) \hat{E}_D^+(t) \right\rangle \tag{2}$$

in which \hat{E}_{C}^{+} and \hat{E}_{D}^{+} are respectively the positive frequency parts of the electric field operator at the output ports C and D of the interferometer. The expectation value of expression (2) is calculated on the state:

$$|\psi_{\rm in}\rangle = |0\rangle + \frac{\beta}{\sqrt{2}} \int d\omega_s d\omega_i \,\phi(\omega_s, \omega_i) \,\hat{a}^{\dagger}_{\omega_s} \hat{a}^{\dagger}_{\omega_i} \,|0\rangle \tag{3}$$

which represents the two photon state produced by the SPDC. The process splits a pump photon at frequency $\tilde{\omega}$ into a signal and an idler photon at frequencies ω_s and ω_i respectively.¹³ In Eq.(3), β is a constant which is proportional to the average number of pump photons in a second.¹⁷ The function $\phi(\omega_s, \omega_i)$ in Eq.(3) is the biphoton wavefunction,¹⁷ which is normalized in such a way that $\int d\omega_s d\omega_i |\phi(\omega_s, \omega_i)|^2 = 1$. In what follows, we will consider the pump field as monochromatic, since its coherence time is assumed to greatly exceeds the one of the down converted photons. As a consequence of this approximation, we can use the energy conservation relation $\tilde{\omega} = \omega_s + \omega_i$ to express the frequency of one photon of the pair as ω , and the frequency of the twin photon as $\tilde{\omega} - \omega$. In this way, the biphoton wavefunction, which we assume to be Gaussian, can be written as:

$$\phi(\omega_s, \omega_i) \simeq \phi(\omega) = \frac{1}{\sqrt[4]{\pi}\sqrt{\sigma}} e^{-\frac{(\omega - \bar{\omega}/2)^2}{2\sigma^2}}$$
(4)

in which σ is the bandwidth of the generated photons.

We can distinguish between two-photon and single-photon-like interference effects by studying Eq.(2) as a function of a relative time delay $\Delta \tau$ between the two MZ arms. Experimentally, we use a heater to vary $\Delta \tau$, and hence we indicate the upper arm with the subscript h (heater) and the lower arm with r (reference). The propagation losses along the two arms are given by γ_h^2 and γ_r^2 respectively. Combining this with the beamsplitter relations (1), we can write the electric fields E_c and E_d as:

$$E_{c}^{-}(t) = r_{1}\gamma_{h}r_{2c}e^{-i2\delta}E_{in}^{-}(t-\Delta\tau) + t_{1}\gamma_{r}t_{2c}E_{in}^{-}(t)$$

$$E_{d}^{-}(t) = t_{1}\gamma_{r}r_{2d}e^{-i\delta}E_{in}^{-}(t) + r_{1}\gamma_{h}t_{2d}e^{-i\delta}E_{in}^{-}(t-\Delta\tau)$$
(5)

where E_{in}^- is the negative frequency part of the input electric field operator at BS1. From these expressions for the electric fields, we can calculate the photon coincidence rate, provided that we also specify the input radiation state $|\Psi_{in}\rangle$. By inserting a completeness relation between $E_c^-(t+\tau)$ and $E_c^+(t+\tau)$ in Eq.(2), and by using Eq.(5) we find:

$$P(\tau, \Delta \tau) = K \Big| \gamma_{hc} \gamma_{rd} e^{-i3\delta} \langle E_{in}^{-}(t) E_{in}^{-}(t + \tau - \Delta \tau) \rangle + + \gamma_{rd} \gamma_{rc} e^{-i\delta} \langle E_{in}^{-}(t) E_{in}^{-}(t + \tau) \rangle + \gamma_{hc} \gamma_{hd} e^{-i3\delta} \langle E_{in}^{-}(t - \Delta \tau) E_{in}^{-}(t + \tau - \Delta \tau) \rangle + \gamma_{hd} \gamma_{rc} e^{-i\delta} \langle E_{in}^{-}(t - \Delta \tau) E_{in}^{-}(t + \tau) \rangle \Big|^{2}$$

$$(6)$$

where the expectation values are now evaluated between the initial state $|\Psi_{in}\rangle$ and the vacuum state $|0\rangle$, i.e $\langle E_{in}^{-}(t)E_{in}^{-}(t')\rangle = \langle \Psi_{in}|E_{in}^{-}(t)E_{in}^{-}(t')|0\rangle$. In Eq.(6) we have introduced the parameters:

$$\gamma_{hc} = r_1 \gamma_h r_{2c} \quad \gamma_{hd} = r_1 \gamma_h t_{2d}$$

$$\gamma_{rc} = t_1 \gamma_r t_{2c} \qquad \gamma_{rd} = t_1 \gamma_r r_{2d}$$
(7)

where the subscript h, r refers to the path along the upper or lower arm of the interferometer respectively, while subscript c, d denotes whether the photon arrives at detector C or D.

The expectation values in Eq.(6) can be evaluated by using the Fourier representation of the negative frequency part of the input electric field:

$$E_{in}^{-}(t) = \int a_{\omega}^{\dagger} e^{i\omega t} d\omega \tag{8}$$

and hence that:

$$\langle E_{in}^{-}(t)E_{in}^{-}(t')\rangle = 2\phi(t-t')e^{i\tilde{\omega}t'}$$
(9)

where $\phi(t)$ is the Fourier transform of $\phi(\omega)$. Substituting Eq.(9) into Eq.(6), we obtain that $P(\tau) \propto |p_{h,h} + p_{r,r} + p_{h,r}^{(R)} + p_{h,r}^{(T)}|^2$, where:

$$p_{h,h} = 2\gamma_{hc}\gamma_{hd} \, e^{-i(\tilde{\omega}\Delta\tau + 3\delta)} e^{-\frac{\sigma^2\tau^2}{2}} \tag{10}$$

$$p_{r,r} = 2\gamma_{rc}\gamma_{rd} e^{-\frac{\sigma^2 \tau^2}{2}} e^{-i\delta}$$
(11)

$$p_{r,h}^{(R)} = 2\gamma_{hc}\gamma_{rd} e^{-i\left(\frac{\tilde{\omega}\Delta\tau}{2} + 3\delta\right)} e^{-\frac{\sigma^2(\tau - \Delta\tau)^2}{2}}$$
(12)

$$p_{r,h}^{(T)} = 2\gamma_{hd}\gamma_{rc} \, e^{-i\left(\frac{\tilde{\omega}\Delta\tau}{2} + \delta\right)} \, e^{-\frac{\sigma^2(\tau + \Delta\tau)^2}{2}} \tag{13}$$

Eqs. (10)-(13) are the transition amplitudes associated with the indistinguishable paths through which the photon pair can travel from the input of BS1 to the photodetectors, as sketched schematically in the grey shaded region of Fig.2. As can be seen, the amplitudes $p_{h,h}$ and $p_{r,r}$ refer to bunching, when both photons are either reflected or transmitted by BS1. On the other hand, the amplitudes $p_{h,r}^{(R)}$ and $p_{h,r}^{(T)}$ describe anti-bunching, when the photon pair is split at BS1. In these cases, the superscript R, T denotes respectively when the photons are both either reflected or transmitted at BS2.

We emphasise that the antibunching paths $(p_{h,r}^{(T,R)})$ are not generally allowed when both input ports of BS1 are excited, due to the Hong-Ou-Mandel effect at the first beamsplitter.¹³ By using only a single input port, we therefore explore a richer interplay of interference effects where the photon pair can travel along both antibunching and bunching paths.

To calculate the coincidence rate, we have to square the sum of all the transition amplitudes in Eqs.(10)-(13), and integrate τ over the coincidence resolving time of the photodetectors T_R . As can be seen from the expressions above, each amplitude vanishes when τ is much greater than the photon coherence time $\tau_c = 1/\sigma$, while for all practical experiments $T_R \gg \tau_c$. Hence, we can effectively extend the integration over τ from $-\infty$ to ∞ , to finally obtain:

$$P(\Delta \tau) = K'(C_1 + C_2 + C_3) \tag{14}$$

where K' is a constant and the three terms on the right hand side are defined as follows:

$$C_1 = \gamma_{hc}^2 \gamma_{rd}^2 + \gamma_{hd}^2 \gamma_{rc}^2 + 2A_{\tilde{\omega}} e^{-\sigma^2 \Delta \tau^2} \cos(2\delta)$$
(15)

$$C_2 = \gamma_{hc}^2 \gamma_{hd}^2 + \gamma_{rc}^2 \gamma_{rd}^2 + 2A_{\tilde{\omega}} \cos(\tilde{\omega}\Delta\tau - 2\delta)$$
(16)

$$C_3 = 2e^{-\frac{\sigma^2 \Delta \tau^2}{4}} \left[A^{(1)}_{\tilde{\omega}/2} \cos\left(\frac{\tilde{\omega} \Delta \tau}{2} + 2\delta\right) + A^{(2)}_{\tilde{\omega}/2} \cos\left(\frac{\tilde{\omega} \Delta \tau}{2}\right) \right]$$
(17)

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where we have introduced the following parameters:

$$A_{\tilde{\omega}} = \gamma_{rc} \gamma_{rd} \gamma_{hc} \gamma_{hd} \tag{18}$$

$$A_{\tilde{\omega}/2}^{(1)} = \gamma_{hc}\gamma_{rc}(\gamma_{rd}^2 + \gamma_{hd}^2) \tag{19}$$

$$A_{\tilde{\omega}/2}^{(2)} = \gamma_{hd}\gamma_{rd}(\gamma_{hc}^2 + \gamma_{rc}^2) \tag{20}$$

It will also be convenient to introduce the power-amplitude coefficient $A_{\tilde{\omega}/2}$ associated with the frequency component at $\tilde{\omega}/2$:

$$A_{\tilde{\omega}/2}^2 = \left(A_{\tilde{\omega}/2}^{(1)}\right)^2 + \left(A_{\tilde{\omega}/2}^{(2)}\right)^2 + 2A_{\tilde{\omega}/2}^{(1)}A_{\tilde{\omega}/2}^{(2)}\cos(2\delta)$$
(21)

In Eqs.(15)-(17), we have grouped into $C_{1,2,3}$ those terms which arise respectively from the interplay of antibunching with antibunching paths; of bunching with bunching paths; and of antibunching with bunching paths. These various combinations are illustrated schematically in the table of Fig.2, where the photon paths are shown in the first gray-shaded row and column, and the resulting terms in $P(\Delta \tau)$ are sketched in the main entries of the table. For example, all terms on the diagonal of this table represent the interference of a two-photon path with itself and hence are independent of the delay $\Delta \tau$ and are depicted as a constant contribution. We now summarise the other main characteristics of our three different groupings of interference terms.



Figure 2. A schematic summarising the interference terms between all the possible paths leading to a coincidence photodetection. The paths are sketched in the panels with a gray shaded background. From left to right, in the uppermost row, we have: $(p_{h,h})$ both photons are reflected in the upper arm, $(p_{r,r})$ both photons are transmitted in the lower arm, $(p_{h,r}^{(R)})$ photons are split by BS1 and reach the detectors by two reflections at BS2, $(p_{h,r}^{(T)})$ photons are split by BS1 and reach the detectors by two transmission at BS2. The transition amplitudes associated with the paths are given in Eq.(10)-(13). The main entries of the table sketch the resulting contribution to the correlation function from the interference of these paths as a function of $\Delta \tau$, where we have integrated in τ as described in the main text. By proceeding from left to right, for example, the panels in the second row represent contributions from $|p_{h,h}|^2$, $p_{h,h}^*p_{r,r}$, $p_{h,h}^{(R)}p_{h,r}^{(R)}$ and $p_{h,h}^*p_{h,r}^{(T)}$.

Firstly, as illustrated in the four bottom-right entries of Fig.2, the interplay of antibunching with antibunching terms in Eq.(15) includes the characteristic Hong-Ou-Mandel dip in the coincidence due to the destructive interference of the two different antibunching paths at the second beamsplitter. This reduction in the coincidence is largest when the time delay between the two arms of the MZ is equal to zero, i.e when we are at the optical contact of the interferometer. Note that here we assume that $\cos(2\delta)$ is negative.

Secondly, we see in the four top-left main panels of Fig.2 that the interaction between the two different bunching paths leads to fringes at a frequency which is doubled with respect to the average frequency $\tilde{\omega}/2$ of

the two photons. As can be seen from Eq.(16), these oscillations persist even when the time delay $\Delta \tau$ exceeds the single-photon coherence time τ_c . These therefore are two-photon interference effects arising from the colourentanglement of the SPDC radiation.

Finally, in the top-right and bottom-left entries of Fig.2, the interplay between the antibunching and bunching paths leads to terms which oscillate at the photon average frequency $\tilde{\omega}/2$. As can be seen from Eq.(17), these terms are damped out as $\Delta \tau$ increases beyond τ_c , as is characteristic of single-photon interference.

The combination of all of these terms into Eq.(14) leads to a complicated coincidence pattern which will, in general, contain features from both the two-photon and single-photon interference. The strength of the different oscillating terms is respectively measured by the terms $A_{\tilde{\omega}}$ and $A_{\tilde{\omega}/2}$ in Eqs.(18)-(21). Their ratio

$$\xi = \frac{A_{\tilde{\omega}/2}}{A_{\tilde{\omega}}} \tag{22}$$

which we call the *unbalancing parameter*, will be used in the following to quantify the relative magnitude of these effects. To understand the different physical regimes, we begin from the simplest case of ideal lossless beamsplitters. Then the relative phase of the reflection and transmission coefficients in the beamsplitters is set by energy conservation as $\delta_1 = \delta_2 = \pi/2$.^{15, 18} Consequently, the unbalancing parameter simplifies to:

$$\xi = \frac{A_{\tilde{\omega}/2}}{A_{\tilde{\omega}}} = \frac{(\gamma_{hc}\gamma_{rd} - \gamma_{rc}\gamma_{hd})(\gamma_{hc}\gamma_{hd} - \gamma_{rc}\gamma_{rd})}{\gamma_{rc}\gamma_{rd}\gamma_{hc}\gamma_{hd}}$$
(23)

and we can straightforwardly consider the two limits of $\xi = 0$ and $\xi \to \infty$.

In the limit when $\xi = 0$, antibunching-bunching interactions undergo complete destructive interference, and the single-particle-like features disappear from the coincidence pattern. This happens if we impose some symmetries on the arm losses or on the beamsplitter coefficients. The simplest such example is when both the beamsplitters are 50 : 50 devices and the two arms have identical loss rates; we then have $\gamma_{hc} = \gamma_{hd} = \gamma_{rc} = \gamma_{rd}$ and $A_{\tilde{\omega}/2} = 0$. Thus, only one frequency is observed when the device is ideally-symmetric, which is consistent with what was found in previous works.^{9,10} We also note that there are three other configurations for which $\xi = 0$:

- 1. If BS2 is balanced, so $\gamma_{hc} = \gamma_{hd}$ and $\gamma_{rd} = \gamma_{rc}$.
- 2. If BS1 is balanced while the arm loss γ_r and γ_h is also equal, so $\gamma_{hc} = \gamma_{rc}$ and $\gamma_{rd} = \gamma_{hd}$.
- 3. If the transmittance from the input to port C along the upper arm is equal to the transmittance to port D along the lower arm (i.e. $\gamma_{rd} = \gamma_{hc}$) or vice-versa ($\gamma_{hd} = \gamma_{rc}$).

In the opposite limit of $\xi \to \infty$, the coincidence rate will show no features of two-photon correlations. This happens when one of the four factors in Eq.(7) is equal to zero. To see this, we consider, for example, $\gamma_{hc} = 0$; then a photon collected at detector C can only have come from the lower MZ arm, providing *which-way* information and destroying any two-photon interference.

As we shall study in the following, values of the unbalancing parameter ξ between $[0, \infty]$ occur when no particular symmetries are imposed. This leads to coincidence patterns where the hallmarks of both one-photon and two-photon interference are present.

3. EXPERIMENTAL VALIDATION OF THE MODEL

To validate the model in Section2, we generated colour-entangled photons near 1550 nm by using a 1 mm long periodically-poled Lithium Niobate crystal and a 775 nm continuous wave pump laser.¹⁴ The quite large banwidth of the generated radiation, which has been measured to be $\approx 300 nm$, is due to the small length of the crystal. By using type-0 SPDC, we obtained co-polarized and collinear signal and idler photons. We implemented a long wavelength pass filter (with a cutoff wavelength of 1500 nm) to reject the pump from the IR light with an isolation higher than 100 dB.

In order to introduce a variable time delay between the two arms of the interferometer, a cylinder of Borosilicate Crown glass (NbK7) was placed in both MZ arms, and, in one arm, the NbK7 was connected to an electric heater.¹⁴ We then used the thermo-optic coefficient of the NbK7 to smoothly vary its refractive index as we changed the temperature. In this way, we were able to induce a variable time delay between the arms as long as 150 fs. This time delay was extracted by assuming a central wavelength of the SPDC radiation of 1550 nm, corresponding to a fringe period of 5.16 fs.

The photons at the output ports of the interferometer were then focused by two lenses onto two InGaAs single photon counting detectors (ID Quantique Id210 and ID Quantique Id201). One detector was used in free running mode (40 μs of deadtime) to detect one photon of the pair. When the other detector was triggered, it was enabled for a gate width of 100 ns. The outputs of the photodiodes were then fed into a Field Programmable Gate Array digital correlator that provided the coincidence rate over a coincidence window of 5 ns. As introduced above, we tuned the parameters in Eq.(7) by the relative transmittance between the beams and the lenses of the photon counters.

The results are displayed in Fig.3, with the experimental data shown in black and fits from Eq.(14) shown in red. Our theoretical model was fitted to the experimental results via a genetic algorithm.¹⁹ This fitting procedure was used to determine, for example, the unbalancing parameter ξ labelling each graph; the fitted parameters found in this way were consistent with the controlled misalignments introduced in the experiment. We also measured an average propagation loss factor from the input port of the MZ to the two detectors of $\approx 7 dB$; this is comparable to the theoretical value of 9 dB found using the model in Eq.(7) with fitted parameters.

In the case $\xi = 0.77$ in Fig.3, we balanced the interferometer in order to suppress antibunching-bunching interactions. The residual component at $\tilde{\omega}/2$, the average single-photon frequency of the downconverted photons, is only due to the beamsplitter losses (a detailed analysis is given in Section 4). The coincidence rate exhibits practically the same oscillating behaviour at frequency $\tilde{\omega}$, the pump frequency, within and outside the coherence time of the single photons. The observed pattern becomes a mixture between bunching-bunching and HOM-like interference. The latter manifests itself as a decrease in the average value of the coincidence counts as we approach the optical contact. The oscillation in the coincidence rate outside the coherence time is a clear manifestation of the correlated or entangled nature of the two-photons state created in the down conversion process.¹⁰ To clearly show that the oscillation at frequency $\tilde{\omega}$ is due to purely second order interference effects, we plot in the inset of Fig.3 (panel $\xi = 1.34$) the coincidence rate for time delays greatly exceeding the single photon coherence time τ_c ($\Delta \tau > 100 fs$). Even if not reported in Fig.3, the very same oscillations outside τ_c are observed regardless of the value of ξ .

As ξ is increased, the pattern changes significantly with respect to the balanced situation, due to the enabling of new interference paths. The case at $\xi = 0.83$ in Fig.3 includes two photon, one photon and Hong Ou Mandel interference effects all in a single coincidence pattern. Indeed, outside the coherence time, the antibunching terms sketched in Fig.2 have vanishing probability, so the interference fringes at $\tilde{\omega}$ are due to purely two photon correlation effects. Within the coherence time of the photon wave packet, instead, the antibunching paths are allowed to interfere together with the bunching ones, creating a mixed pattern in which single particle interference at $\tilde{\omega}/2$ and two particle one at $\tilde{\omega}$ coexist. The HOM effect again can be seen as the decrease of the average coincidences within the coherence time. In general the higher is the unbalancing between the arms, the higher is the suppression of the two-photon contribution at $\tilde{\omega}$ and, at the same time, the higher the visibility of the single-photon component at $\tilde{\omega}/2$.

For all three values of the unbalancing parameter in Fig.3, we observe a very good agreement between the experimental results and our fitted theoretical model.



Figure 3. Measured coincidence rates for different values of the unbalancing parameter ξ for SPDC light, taken from Ref.¹⁴ The solid red curves are fits from Eq. (14), while black scatter points are experimental data. The inset shown in the panel $\xi = 1.34$ shows the coincidence rate for time delays larger than the single photon coherence time. The reported value of ξ is taken from the simulation. A value of $\delta \approx 0.87 \frac{\pi}{2}$ has been used, which is compatible with a measured beamsplitter loss of $\sim 22\%$

4. LOSSY BEAMSPLITTER

Remarkably, we found out that the experimental data presented in Section 3 could not be fitted with a phase of $\delta = \pi/2$, corresponding to lossless beamsplitters. For a lossy beamsplitter, we follow the derivation of Barnett et. al.¹⁵ who showed that in general the complex transmittance t and reflectance r satisfy:

$$|t|^2 + |r|^2 \leq 1 \tag{24}$$

where the equality holds for a lossless device. This can be interpreted as the probability of survival for a single photon incident on the beam splitter, as this photon may be now be absorbed instead of transmitted or reflected. Considering incoming classical or coherent fields of equal or opposite amplitude, it can be shown further that:¹⁵

$$|tr^* + t^*r| \leq 1 - |r|^2 - |t|^2 \tag{25}$$

ensuring that the total output intensity is less than or equal to that at the input. If we assume for simplicity $t^* = t = \sqrt{\frac{\chi}{2}}$ and $r = te^{-i\delta}$ to describe a balanced BS where the photon has an intrinsic survival probability χ , the constraint on the relative phase between the reflection and transmission coefficients is:

$$|\cos\delta| \le (1/\chi - 1) \tag{26}$$

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Figure 4. The theoretical coincidence rate in Eq.(6) is plotted against the interferometer time delay $\Delta \tau$ for different values of the beamsplitter phase δ . The parameters of the simulation are the same used for the solid red curve in Fig.3 for $\xi = 1.34$.

When $\chi = 1$ (i.e. for a lossless beamsplitter), this sets $\delta = \pm \pi/2$. When $\chi < 1$, the phase δ can also be higher or lower $\pi/2$, with important consequences. Firstly, we notice from Eq.(15) that if $\delta \neq \pi/2$, the magnitude of HOM dip will be smaller as the antibunching paths acquire different phases upon exiting BS2 and so no longer completely destructively interfere. This is the reduction in the visibility of the Hong-Ou-Mandel effect for a lossy beamsplitter with non-orthogonal reflection and transmission coefficients as predicted in.¹⁵

In the experimental setup described in Section 3, we measure a BS value of $\chi = (0.78 \pm 0.4)$. Then, assuming the beamsplitter is balanced (i.e. |r| = |t|), the range of δ becomes:

$$0.82 \frac{\pi}{2} \lesssim |\delta| \lesssim 1.12 \frac{\pi}{2} \tag{27}$$

While this may at first seem a small difference from $\delta = \pi/2$, it is enough to dramatically affect the appearance of the coincidence pattern, as can be appreciated in Fig.4. We change the phase here from $\delta = \pi/2$ to $\delta = (0.87)\pi/2$, which corresponds to the phase found from our fit to experimental results in Section 3 and it is compatible with the measured beamsplitter losses in Eq.(27). Furthermore we can see that while the condition for $\xi \to \infty$ is not affected by the beamsplitter phase, the limit of $\xi = 0$ is. We can understand this directly from Eq.(21), by nothing that if $\delta \neq \pi/2$, there will never be a complete cancellation of the antibunching-bunching term. However, it can be shown that this term is still minimised by the ideal symmetric configuration where $\gamma_{hc} = \gamma_{hd} = \gamma_{rc} = \gamma_{rd}$. As we can see this phase change affects many of the qualitative features in these patterns, including the envelope, behaviour around the optical contact, and relative position of the peaks from the different frequency oscillations. Again we emphasise that this remarkable phase sensitivity arises from the rich interplay of multiple interference terms, going beyond previous experiments.^{9,10}

5. CONCLUSIONS

In this work, we revisited the Mach-Zehnder interferometer removing all elements of ideality which characterized previous experiments in quantum optics. We presented a theoretical model for fourth-order interference patterns when colour-entangled photon pairs are sent into the same input port of the MZ interferometer, i.e when the device is asymmetrically excited. We showed that this configuration leads to a much wider-range of phenomena than previously studied in one experiment, as it allowed us to observe the hallmarks of both one-photon and two-photon interference within the same coincidence pattern. We discussed how the relative importance of these effects can be tuned experimentally, and highlighted the sensitivity of these results to additional phase effects from beamsplitter losses. We showed that our theoretical model fitted experimental results with very good agreement.

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