Intermode reactive coupling induced by waveguide-resonator interaction

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We report on a joint theoretical and experimental study of an integrated photonic device consisting of a single-mode waveguide vertically coupled to a disk-shaped microresonator. Starting from the general theory of open systems, we show how the presence of a neighboring waveguide induces a reactive intermode coupling in the resonator, analogous to an off-diagonal Lamb shift in atomic physics. Observable consequences of this coupling manifest as peculiar Fano line shapes in the waveguide transmission spectra. The theoretical predictions are validated by full vectorial three-dimensional finite-element numerical simulations and are confirmed by the experiments.

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I. INTRODUCTION

The study of the consequences of coupling a physical system to an environment constitutes the central problem in the theory of open systems [1]. This coupling, on one hand, allows the system to dissipate energy through active decay channels. On the other hand, its reactive component leads to a shift of energy levels and oscillation frequencies of the system. Most celebrated examples of this physics involve an atom coupled to the bath of electromagnetic modes [2], namely, the (dissipative) spontaneous emission of photons from an excited state [3–5] and the (reactive) Lamb shift of transition frequencies [6–8].

Pioneering experimental studies in late 1970s [9] showed that destructive interference of different decay paths, leading to the same final continuum, can suppress absorption by a multilevel atom via the so-called coherent population trapping [10] and electromagnetically induced transparency (EIT) [11,12] mechanisms. While originally these phenomena were discovered in the atomic physics context, a continuous interest has been devoted to analogous effects in solid-state systems [13], quantum billiards [14–16], photonic devices [17–23], and, very recently, optomechanical systems [24]. Although in most experiments only the dissipative features are affected by the interference, the theory predicts that a similar phenomenon should also occur for the reactive ones [1].

In photonics, the presence of a waveguide in the vicinity of a resonator activates new radiative decay channels for the resonator modes via emission of light into the waveguide mode [25–27]. The corresponding reactive effect is a shift of the resonator mode frequencies, which can be interpreted as the photonic analog of the atomic Lamb shift. In this paper, we report on a joint theoretical and experimental study of a photonic device in which pairs of modes of very similar frequencies are coupled simultaneously to the same waveguide mode. Both the dissipative and the reactive couplings of the cavity modes to the waveguide turn out to be affected by interference phenomena between the two modes, which can be summarized as environment-induced intermode couplings: in the atomic analogy, the dissipative component gives a coherent population trapping phenomenon, while the reactive one produces a sort of *off-diagonal Lamb shift*. In this work, we will show that the consideration of both of these coupling terms is necessary in order to explain the peculiar Fano interference line shapes experimentally observed in the transmission spectra of single resonators.

The paper is organized as follows. In Sec. II we introduce the integrated photonic system under consideration, formed by a whispering-gallery disk microresonator vertically coupled to a single-mode waveguide. In Sec. III we first introduce the theoretical model which is used to study the transmission through the waveguide-cavity system in the general case of an arbitrary number of modes. This model is then used to obtain analytical insight on the asymmetric Fano line shapes that appear as soon as two (or more) cavity modes are simultaneously excited. In Sec. IV we present the results of our full three-dimensional (3D) numerical calculations. Experimental data, fully confirming our theoretical predictions, are illustrated in Sec. V. Conclusions are finally drawn in Sec. VI.

II. THE VERTICALLY COUPLED RESONATOR-WAVEGUIDE SYSTEM

The system under consideration consists of a thin microdisk resonator vertically coupled to an integrated single-mode waveguide located below the disk [Fig. 1(a)]. In contrast to the traditional lateral coupling geometry, where typically only the most external first radial mode family (RMF) experiences an appreciable coupling to the waveguide, the vertical coupling geometry allows for an independent lateral and vertical positioning of the waveguide, permitting thus the coupling to the different mode families to be freely tuned, in particular that to the more internal ones [28,29]. Since these latter typically have lower intrinsic quality factors, the vertical coupling geometry is crucial to our experiments, as it allows for several RMFs to be simultaneously close to critical coupling and, therefore, visible in transmission spectra.

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FIG. 1. (Color online) (a) A sketch of the microphotonic device. (b) The intensity profile of the first and second radial mode families (RMFs) of the resonator (top and middle panels) and of the waveguide mode (bottom panel). The (blue) curves show the cuts of the intensity profile, and the labels indicate the different materials. (c) (d) Results of *ab initio* numerical calculations for (c) the radiative decay rate ratio $\Gamma_{22}^{rad}/\Gamma_{11}^{rad}$ and (d) the frequency shifts of the first (Δ_{11} , top) and the second (Δ_{22} , bottom) radial family modes as functions of the waveguide position. The open circles indicate the waveguide position for the fabricated 40- μ m-diameter resonator.

The remarkable tunability of the waveguide-resonator coupling is numerically illustrated using ab initio finiteelement numerical simulations performed using a commercial 3D full-wave finite-element method (FEM) software [30]. The results for the frequencies and damping rates of the different resonator modes are shown in Figs. 1(c) and 1(d): numerically, they have been obtained from the eigenmodes of the electromagnetic wave equation with suitable absorbing boundary conditions. In order to minimize the contribution from mode-coupling terms, at this stage we have focused on a case where the lowest two RMFs of the resonator are spectrally separated, so that the numerical eigenfrequencies provide the frequencies and linewidths of the two modes. A brief discussion of the physical meaning of the numerical eigenfrequencies when the two resonator modes are spectrally close is given in the Appendix. In all considered cases, the numerical simulations confirm the experimental observation that the strength of the backscattering into counterpropagating modes by the waveguide is negligible as compared to the decay rates [31].

The ratio of the radiative decay rates of the two modes shown in Fig. 1(c) is proportional to the relative intensity of their coupling to the waveguide: as expected, this value is the largest when the lateral position of the waveguide matches the main lobe of the the second RMF [Fig. 1(b) middle panel]. The photonic analog of the atomic Lamb shift for (independent) cavity modes is illustrated in Fig. 1(d): the frequency shifts Δ_{11} and Δ_{22} of the two modes are measured from the *bare* frequencies of the modes when the waveguide is far apart from the resonator. While in the atomic case the calculation of the Lamb shift, originating from photon emission and reabsorption processes, requires sophisticated techniques and a careful handling of UV divergences [2], in the photonic case one typically has a redshift of all modes when a generic dielectric material is brought close to a resonator [25].

III. ANALYTICAL THEORY

In this section, we present our theoretical predictions for the transmittivity of the waveguide-resonator device. While the theoretical approach is fully general and can be applied to arbitrary multimode cases, much of our attention will be focused on the simplest two-mode model, which already captures the interesting new physics originating from the intermode coupling terms. In the experiment, the two resonator modes circulate in the same direction around the disk but belong to different radial families. Future work will deal with more complex configurations where many modes are simultaneously close to resonance and/or backscattering processes induce significant couplings to counterpropagating modes.

A. The input-output model

The transmission of a waveguide coupled to a resonator can be described by generalizing the input-output theory of optical cavities [32] to the multimode case. In the present two-mode case, the equation of motion for the field amplitudes $\alpha_{j=1,2}$ can be written as

$$i\frac{d\alpha_{j}}{dt} = \left[\omega_{j}^{o} + \Delta_{jj} - i\frac{\gamma_{j}^{\text{nr}} + \Gamma_{jj}^{\text{rad}}}{2}\right]\alpha_{j} + \left(\Delta_{12} - i\frac{\Gamma_{12}^{\text{rad}}}{2}\right)\alpha_{3\text{-}j} + \bar{g}_{j}E_{\text{inc}}(t).$$
(1)

In the absence of the waveguide, the two modes oscillate independently of each other at a bare frequency ω_j^o and have an intrinsic, nonradiative decay rate γ_j^{nr} . The incident field, which propagates along the waveguide and drives the resonator, is described in the last term in Eq. (1). The coupling amplitude of the driven waveguide mode to the j = 1,2 resonator mode is quantified by the \bar{g}_j coefficients. In the following, we focus on a monochromatic incident field $E_{\text{inc}}(t) = E_{\text{inc}} e^{-i\omega_{\text{inc}}t}$.

The effect of the waveguide on the cavity mode oscillation is included in the motion equation (1) via the Hermitian Γ^{rad} and Δ matrices, for which formal application of the theory of open systems within the Markov approximation [1] provides the general expression

$$\Delta_{jl} + i \frac{\Gamma_{jl}^{\text{rad}}}{2} = \int \frac{dK}{2\pi} \sum_{\beta} \frac{g_{\beta,j}^*(K) g_{\beta,l}(K)}{\omega_{\text{inc}} - \Omega_{\beta}(K) - i0^+}, \quad (2)$$

in terms of the coupling amplitude $g_{\beta,j}(K)$ of the *j*th resonator mode to that of the waveguide of longitudinal wave vector *K*, mode index β , and frequency $\Omega_{\beta}(K)$. For single-mode waveguides, Γ^{rad} is determined by the single propagating mode for which $\Omega_{\beta}(K) = \omega_{\text{inc}}$ and we can take $g_{\beta,j}(K) = \bar{g}_j$ real and positive. This imposes the requirement that the Γ_{12}^{rad} coefficient, typically responsible for EIT-like interference effects in the atomic context, is related to the radiative linewidths Γ_{jj}^{rad} by $\Gamma_{12}^{\text{rad}} = \sqrt{\Gamma_{11}^{\text{rad}}\Gamma_{22}^{\text{rad}}}$.

Even though a quantitative estimation of Δ using Eq. (2) is in most cases impractical as it involves a sum over all (both guided and nonguided) waveguide modes, this equation provides an intuitive picture of the underlying process: the diagonal and off-diagonal terms originate from the virtual emission of a photon from a resonator mode and its immediate recapture by the same or another mode, respectively. From a qualitative point of view, while the diagonal terms are typically $\Delta_{jj} < 0$, we are unable to invoke any general argument to determine the nondiagonal Δ_{12} . A similar intermode coupling term was mentioned in [20] starting from a coupled-mode approach. Below, we will see how a real $\Delta_{12} > 0$ is needed to reproduce the experimental data and we will point out some unexpected features due to this term.

B. The transmittivity of two-mode resonators

In our model, the waveguide transmission reads

$$T(\omega_{\rm inc}) = |E_{\rm tr}/E_{\rm inc}|^2 = |1 - i \ \rho \sum_{j=1,2} \bar{g}_j^* \bar{\alpha}_j / E_{\rm inc}|^2, \quad (3)$$

in terms of the stationary solution $\bar{\alpha}_j$ of the motion equations (1) and the density of states $\rho = |dK/d\Omega|$ in the waveguide. When the waveguide is effectively coupled to one resonator mode only, a typical resonant transmission dip is recovered: under-, critical-, and overcoupling regimes are found depending on whether Γ_{11}^{rad} is lower than, equal



FIG. 2. (Color online) Analytically calculated transmission spectra in different regimes and for different detunings $\delta = \omega_2 - \omega_1$. (a) Both modes are undercoupled, $\Gamma_{11,22}^{rad}/\gamma_{1,2}^{nr} = 0.25$ and $\Delta_{12} = 0$. (b) Both modes are overcoupled, $\Gamma_{11,22}^{rad}/\gamma_{1,2}^{nr} = 4$ and $\Delta_{12} = 0$. (c) Narrow (broad) mode is undercoupled (overcoupled), $\Gamma_{11}^{rad}/\gamma_1^{nr} = 0.125$ ($\Gamma_{22}^{rad}/\gamma_2^{nr} = 2$) and $\Delta_{12}/\gamma_2^{nr} = 0.6$. The ratio $\gamma_1^{nr}/\gamma_2^{nr} = 0.1$ (a-b) and 0.16 (c).

to, or larger than γ_1^{nr} . A brief analytical discussion of this well-known [25–27] single-mode physics is given in the next section.

The much richer phenomenology that occurs in the twomode case is illustrated in Fig. 2. Interesting features manifest clearly when both j = 1,2 resonator modes are close to criticality, $\gamma_j^{nr} \approx \Gamma_{jj}^{rad}$. In Fig. 2(a), we show a case where both modes are slightly undercoupled $\Gamma_{jj}^{rad} \lesssim \gamma_j^{nr}$ and the off-diagonal reactive coupling vanishes, $\Delta_{12} = 0$. Each mode then manifests as a transmission dip in the spectrum centered at a frequency $\omega_j = \omega_j^o + \Delta_{jj}$ that includes the diagonal shift Δ_{jj} . Note that the the first RMF is much narrower than the other since $\gamma_1^{nr} \ll \gamma_2^{nr}$. Comparing the different rows of the figure, we notice that scanning the relative detuning of the two modes $\delta = \omega_2 - \omega_1$ results in a simple, interference-free superposition of the two dips. Even in this simplest case, a correct inclusion of Γ_{12}^{rad} is, however, essential to avoid the appearance of nonphysical features in the calculations, such as $T(\omega) > 1$.

Figure 2(b) shows the case of slightly overcoupled modes $\Gamma_{jj}^{rad} \gtrsim \gamma_j^{nr}$, still with $\Delta_{12} = 0$. Now, marked interference features start to appear due to the off-diagonal dissipative coupling Γ_{12}^{rad} , and the doublets of peaks acquire a complicated structure. In particular, the narrow dip, normally visible at ω_1 (first and seventh rows), is replaced by a complex Fano-like line shape [2,16,33] (third and fifth) for moderate detunings, and even reverses its sign into a transmitting EIT feature in the resonant $\delta = 0$ case (fourth row). Experimental observations of this physics were recently reported in [18–20].

Finally, the dramatic effect of the off-diagonal reactive coupling $\Delta_{12} > 0$ is shown in Fig. 2(c). As the most visible general feature, the spectrum is no longer symmetric under a change in the sign of δ , and the spectral feature due to the narrow mode is more clearly visible than one would expect given its deep undercoupling condition. With respect to the $\Delta_{12} = 0$ case shown in Fig. 2(b), the narrow Fano feature

has a reversed sign for moderate detunings (third and fifth rows). Furthermore, it is suppressed in a finite detuning range (sixth row). An analytical explanation of this unexpected effect will be given in the next section in terms of the destructive interference of the direct excitation of mode 1 from the waveguide and its two-step excitation via mode 2 by the off-diagonal terms of Δ and Γ . The two paths almost cancel out around $\delta \simeq \Delta_{12} \sqrt{\Gamma_{22}^{rad} / \Gamma_{11}^{rad}}$.

C. Analytical study of the Fano line shapes

Starting from the analytical expression (3) for the transmittivity, in this section we will propose an analytical explanation for the unexpected features found in the previous section for the transmission spectra of two-mode cavities. As a first step, it is useful to isolate in the motion equation (1) the contribution Δ_{jl}^{rad} of the single propagating mode of the waveguide to the reactive matrix Δ_{jl} , $\Delta_{jl} = \Delta_{jl}^{\text{rad}} + \Delta_{jl}^{\text{other}}$. Introducing (real) relative weights $\eta_{1,2}$ with $\eta_1^2 + \eta_2^2 = 1$ for the coupling amplitude of the two resonator modes to the single propagating waveguide mode, one can then write

$$\Gamma_{jl}^{\rm rad} = \eta_j \eta_l \bar{\Gamma}^{\rm rad},\tag{4}$$

$$\Delta_{jl}^{\rm rad} = \eta_j \eta_l \bar{\Delta}^{\rm rad} \tag{5}$$

in terms of an overall radiative dissipation rate $\bar{\Gamma}^{rad}$ and an overall frequency shift $\bar{\Delta}^{rad}$. Based on experimental input, we assume that all other waveguide modes do not contribute to the off-diagonal coupling $\Delta_{12}^{other} = 0$ and we reabsorb the diagonal terms into $\omega_{1,2}^{b} = \omega_{1,2}^{o} + \Delta_{11,22}^{other}$.

In this notation, we then have

$$\begin{pmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} E_{\text{inc}}$$
(6)

with

$$\mathbf{M} = \begin{pmatrix} \delta_1 & a \\ a & \delta_2 \end{pmatrix} \tag{7}$$

and

$$\delta_1 = \omega_{\rm inc} - \omega_1^{\rm b} + \frac{i}{2} \left(\gamma_1^{\rm nr} + \eta_1^2 \bar{\Gamma}^{\rm rad} \right) - \eta_1^2 \bar{\Delta}^{\rm rad}, \qquad (8)$$

$$\delta_2 = \omega_{\rm inc} - \omega_2^{\rm b} + \frac{i}{2} \left(\gamma_2^{\rm nr} + \eta_2^2 \bar{\Gamma}^{\rm rad} \right) - \eta_2^2 \bar{\Delta}^{\rm rad}, \qquad (9)$$

$$a = i\eta_1\eta_2 \left(\frac{\bar{\Gamma}^{\rm rad}}{2} + i\bar{\Delta}^{\rm rad}\right). \tag{10}$$

Inserting this form into the expression for the transmission amplitude

$$t(\omega_{\rm inc}) = 1 - i\bar{\Gamma}^{\rm rad}(\eta_1\bar{\alpha}_1 + \eta_2\bar{\alpha}_2), \qquad (11)$$

and performing a bit of algebraic manipulation, one obtains a compact expression

$$t(\omega_{\rm inc}) = 1 - i \frac{\bar{\Gamma}^{\rm rad} \eta_2^2}{\delta_2} - i \frac{\bar{\Gamma}^{\rm rad} \eta_1^2}{\delta_1 - a^2/\delta_2} \left(1 - \frac{a}{\delta_2} \frac{\eta_2}{\eta_1}\right)^2 \quad (12)$$

for the transmission amplitude.

In the limiting case where a single mode is coupled to the waveguide, e.g., the second one, we can set $\eta_1 = 0$ and η_2 and the last formula reduces to the usual form [27] of the transmission amplitude for a single-mode ring resonator,

$$t(\omega_{\rm inc}) = 1 - i \frac{\bar{\Gamma}^{\rm rad}}{\omega_{\rm inc} - (\omega_2^{\rm b} + \bar{\Delta}^{\rm rad}) + \frac{i}{2} (\gamma_2^{\rm nr} + \bar{\Gamma}^{\rm rad})}, \quad (13)$$

on resonance with the (shifted) mode frequency $\omega_{\rm inc} = \omega_2^{\rm b} + \bar{\Delta}^{\rm rad}$, one in particular has

$$t_{\rm res} = \frac{\gamma_2^{\rm nr} - \bar{\Gamma}^{\rm rad}}{\gamma_2^{\rm nr} + \bar{\Gamma}^{\rm rad}}.$$
 (14)

For $\bar{\Gamma}^{rad} < \gamma_2^{nr} (\bar{\Gamma}^{rad} > \gamma_2^{nr})$ one has the under- (over-) coupling regime and the transmission dip is partial, while it is complete for critical coupling $\bar{\Gamma}^{rad} = \gamma_2^{nr}$.

We now go back to the general expression for the transmittivity Eq. (12): the second term proportional to $1/\delta_2$ describes the broad transmission dip due to the broader mode 2 and the third term involving δ_1 describes the narrow feature due to mode 1. Here, while the frequency shift of the feature given by the a^2/δ_2 term in the denominator does not appear to play a qualitatively important role, the overall strength of the feature is dramatically modified by the square of the (slowly varying) factor

$$F_{1} = 1 - \frac{a}{\delta_{2}} \frac{\eta_{2}}{\eta_{1}}$$

= $\frac{\omega_{\text{inc}} - \omega_{2}^{\text{b}} + \frac{i}{2} \gamma_{2}^{\text{nr}}}{\omega_{\text{inc}} - (\omega_{2}^{\text{b}} + \eta_{2}^{2} \bar{\Delta}_{\text{rad}}) + \frac{i}{2} (\gamma_{2}^{\text{nr}} + \eta_{2}^{2} \bar{\Gamma}_{\text{rad}})}.$ (15)

Following the experiments, we focus our attention on the case where mode 1 is intrinsically narrower than mode 2, $\gamma_1^{nr} \ll \gamma_2^{nr}$. For a very large off-diagonal coupling $\bar{\Delta}_{rad} \gg \gamma_2^{nr}$, $\bar{\Gamma}_{rad}$, in an extended neighborhood of the resonance with mode 2 (that is, for $\omega_{inc} \approx \omega_2^b + \eta_2^2 \bar{\Delta}_{rad}$), this factor can be much larger than 1 in modulus,

$$F_1 \approx \frac{\eta_2^2 \bar{\Delta}_{\rm rad} + \frac{i}{2} \gamma_2^{\rm nr}}{\frac{i}{2} \left(\gamma_2^{\rm nr} + \eta_2^2 \bar{\Gamma}_{\rm rad} \right)} \simeq \frac{\bar{\Delta}_{\rm rad}}{i \bar{\Gamma}_{\rm rad}},\tag{16}$$

which explains why mode 1 is clearly visible in the spectra even in the deep undercoupling regime.

On the other hand, in the vicinity of the bare mode 2 frequency $\omega_{\text{inc}} \simeq \omega_2^{\text{b}}$, this same factor becomes very small,

$$F_1 \approx \frac{\gamma_2^{\rm nr}}{\gamma_2^{\rm nr} + \eta_2^2 \bar{\Gamma}_{\rm rad} + 2i\eta_2^2 \bar{\Delta}_{\rm rad}} \simeq \frac{\gamma_2^{\rm nr}}{2i\bar{\Delta}_{\rm rad}}.$$
 (17)

This simple fact explains the remarkable suppression of the Fano feature observed in both the theory and the experiments for some specific values of the detuning between the two modes.

The frequency region where the suppression is strongest coincides with the (typically unobserved) bare frequency ω_2^b of mode 2. However, the corresponding $\delta_0 = \omega_2^b - \omega_2 \simeq \Delta_{22}^{rad}$ can be related to the (can be fitted on the spectra) coupling term Δ_{12}^{rad} using the fact that the Δ_{jl}^{rad} and the Γ_{jl}^{rad} have the

same dependence on the η_i 's. In particular,

$$\frac{\Delta_{22}^{\text{rad}}}{\Delta_{12}^{\text{rad}}} = \frac{\Gamma_{22}^{\text{rad}}}{\Gamma_{12}^{\text{rad}}} = \sqrt{\frac{\Gamma_{22}^{\text{rad}}}{\Gamma_{11}^{\text{rad}}}},$$
(18)

where we have made use of $\Gamma_{12}^{\text{rad}} = \sqrt{\Gamma_{11}^{\text{rad}}\Gamma_{22}^{\text{rad}}}$ to write the quantity Γ_{12}^{rad} in terms of quantities like Γ_{11}^{rad} and Γ_{22}^{rad} that can be directly extracted from the spectra when the two radial family modes are well separated in frequency.

The factor (15) controlling the suppression effect has a simple physical interpretation in terms of destructive interference of the different processes leading to the excitation of mode 1, namely, its direct excitation from the waveguide (with relative amplitude η_1) and its excitation mediated by mode 2 [with relative amplitude $(\Delta_{12}^{rad} - i\Gamma_{12}^{rad}/2)\delta_2^{-1}\eta_2$]: when the two amplitudes compensate to (almost) zero, the mode 1 remains always empty and is no longer visible in the transmission spectra. When F_1 is significant, its modulus determines the strength of the mode 1 feature and its phase determines the asymmetrical shape of the Fano interference profile [2].

IV. NUMERICAL SIMULATIONS

Further support for the predictions of the analytical model presented in the previous section is provided by ab initio finite-element numerical simulations. The detailed geometrical shape and material composition of the resonator-waveguide system are taken into account in the simulations. However, in order to keep the amount of high-bandwidth memory needed for such a 3D model under reasonable limits, we restricted the simulation domain to the portion of the resonator-waveguide system where the electromagnetic field is really significant. In particular, the inner part of the microdisk has been neglected without a significant loss in calculation accuracy. The extinction coefficient of the material and the boundary limits have been chosen to reproduce the experimentally observed decay rates of the different radial family modes. We have also numerically ensured that a negligible intensity of backscattered light is produced by the numerical mesh.

In contrast to the resonator eigenmodes studied in Sec. II, here we have concentrated on the system transmittivity: First, we solved for the field profiles and the propagation constants at the input and output ports of the waveguide. Then the electric field of the whole geometry was obtained by means of a stationary-state solver in the frequency domain. Finally the transmittance was obtained by evaluating the scattering matrix relating the fields at the input and output ports of the waveguide. The slightly different free spectral range of the different RMFs allowed us to scan the relative detuning of the interfering modes by looking at pairs of quasiresonant modes with different azimuthal quantum numbers.

Examples of spectra for different detunings are shown in the panels of Fig. 3(a). The qualitative agreement with the predictions of Eq. (2) in the $\Delta_{12} > 0$ regime is remarkable: the Fano-like feature is clearly visible with the correct sign for generic detunings (first to fourth rows) and disappears completely in a well-defined range of δ 's (lowest row). The three (A,B,C) panels in Fig. 3(b) show horizontal cuts of the field intensities in the resonator and in the waveguide at three



FIG. 3. (Color online) (a) Numerically calculated spectra for different detunings between the two radial family modes. The azimuthal mode order of the two resonances is reported in each graph. (b) Planar cuts of the intensity profile inside the resonator and inside the waveguide at the frequencies indicated as A, B, and C in (a). (c) The polygonal shape of the spatially oscillating mode profile is explained via an interference between the fields $E_1 = E_1^0 \cos(\theta M_1)$ and $E_2 = E_2^0 \cos(\theta M_2)$ of two modes with different azimuthal mode numbers M_1 and M_2 . (d) This interference pattern is illustrated as a function of the azimuthal angle θ and the number M_2 of the second-order radial mode for a fixed $M_1 = 125$ of the first-order radial mode: as expected, the number of polygon vertices is determined by the difference $|M_2 - M_1|$.

different incident frequencies across the Fano-like feature as indicated in the second panel of Fig. 3(a). While the excitation at the A (C) point is concentrated in the second (first) mode, interference between the two modes is responsible for the snaky shape of the intracavity intensity distribution at the intermediate point B. As expected [see Figs. 3(c) and 3(d)], the number of spatial oscillations is determined by the difference in azimuthal quantum numbers of the two resonator modes. We note that a similar picture is observed for the other spectra shown in different panels of Fig. 3(a).

V. EXPERIMENTS

In the previous sections we predicted and discussed interesting Fano features in the transmission spectra of a vertically coupled resonator-waveguide device: the analytical



FIG. 4. (Color online) (Left) An optical photograph of the fabricated 40- μ m-diameter microresonator and (a) the measured transmission spectrum as a function of the absolute incident frequency. The azimuthal mode numbers M_1 and M_2 are indicated next to the different firstand second-order radial modes. The two modal families have slightly different free spectral ranges of FSR₁ \approx 1.236 THz and FSR₂ \approx 1.256 THz. (b)–(g) Blow-ups of the regions marked in gray in (a). In each panel, the relative frequency is measured from the broader second family resonance. Red lines show fits to the spectra using the analytical model.

predictions of Sec. III were validated by *ab initio* numerical simulations in Sec. IV. We now proceed with the presentation of our experiments and the comparison of their results with the theoretical prediction.

A. Sample preparation and optical characterization

The samples studied in this work were realized using standard silicon microfabrication tools, as detailed in our previous works [28,29]. The process starts with growing a 3- μ m-thick thermal silicon dioxide cladding on top of a silicon wafer and is followed by plasma-enhanced chemical vapor deposition (PECVD) growth of a 300 nm silicon oxynitride (SiON) layer. The waveguide structures are lithographically patterned and transferred to the SiON layer using a reactive-ion etching step. Next, a borophosphosilicate glass is deposited and flowed again at 1050 °C to form a planar top cladding over the waveguides. Silicon nitride (SiN_x) resonators were realized in a 400-nm-thick layer deposited using PECVD and defined through a combination of lithographic and dry etching steps.

The transmittivity was measured in a standard waveguide transmission setup using a near-infrared tunable laser butt coupled through the waveguide facet using lensed optical fibers. In order to ensure accurate and stable alignment conditions, the positions of the lensed fibers were controlled using closed-loop three-axis piezoelectric stages. The signal polarization was controlled at the waveguide input and analyzed at its output. The transmitted signal intensity was recorded with an InGaAs photodiode.

B. Results and discussion

In the experiments, we looked at pairs of quasiresonant modes originating from different radial families in microdisk resonators coupled vertically to dielectric waveguides. The spatial position of the waveguide with respect to the resonator edge is indicated by open dots in Figs. 1(c) and 1(d). The experimental transmission spectrum through the waveguide for a microdisk of radius $R = 40 \ \mu m$ is shown in Fig. 4(a). It consists of a sequence of doublets originating from the first (narrow features) and second (broader features) RMFs, which have slightly different free spectral ranges [31]. This last permits one RMF to be swept across the other as the azimuthal order of the underlying modes is increased, and the doublets' structure is correspondingly changed. In the bottom panels Figs. 4(b)-4(g), zoomed views of the different doublets are shown: to facilitate comparison, in each of these panels the central frequency is located at the broader second family resonance (i.e., at $\omega_2 = \omega_2^o + \Delta_{22}$ in the analytical model). These spectra are in excellent qualitative agreement with the predictions of the numerical simulations shown in Fig. 3: the Fano-like feature has the correct sign and is visible for generic detunings except for a small range of values where it completely disappears [Fig. 4(g)]. Moreover, the experimental data are successfully fitted by the analytical model [red curves in Figs. 4(b)–(g)]. To further appreciate the agreement with theory, a pair of color-map plots, summarizing the experimental findings of Fig. 4 compared to the analytical prediction, are shown in Fig. 5.



FIG. 5. (Color online) (a) Color-map plot merging six experimental transmission spectra of the 40 μ m resonator, shown in Figs. 4(b)–4(g). (b) Color-map plot of the analytical prediction (3) for the transmittivity of a two-mode cavity using globally optimized parameters.



FIG. 6. (Color online) (a) Color map merging 21 experimental transmission spectra (indicated as S1–S21) for a 50 μ m resonator. On each row, the relative frequency is measured from the narrow mode frequency. (b) Analytic prediction for $T(\omega)$ using a three-mode extension of the model with optimized global parameters. (c) Selected examples of spectra. (d), (e) System parameters obtained by independently fitting each experimental spectrum with the analytical model.

The generality of our observations has been confirmed by repeating the experiment on a larger $R = 50 \ \mu m$ resonator in which the Fano interference takes place between the first and the third RMFs. The measured transmission spectra are shown in Figs. 6(a) and 6(c) for different values of the relative detuning of the quasiresonant pairs of modes. The crossing of the two families again leads to Fano interference profiles, and the narrow feature disappears in a specific range of detunings (spectrum S9). Furthermore, the experimental results successfully compare to the prediction of the analytical model, generalized to three modes [Fig. 6(b)].

Finally, Figs. 6(d) and 6(e) summarize the fit parameters for both 40 μ m and 50 μ m resonators. Despite the total independence of the fitting procedures performed on each spectrum, a smooth dependence of all fit parameters on the azimuthal mode number is observed. As expected, the scan of the azimuthal quantum number varies the mode detuning without affecting the other system parameters. From the top graph we notice that in both cases the first family modes are undercoupled to the waveguide, while the second and third family modes are very close to critical coupling. As stated in the theoretical section, this combination of couplings is crucial for a neat observation of the Fano feature. Finally, Fig. 6(e) shows that the fitted value of the off-diagonal reactive coupling Δ_{12} is always around 15 GHz.

VI. SUMMARY AND CONCLUSIONS

To summarize, in this work we have reported a joint theoretical and experimental study of a microdisk resonator vertically coupled to a single-mode waveguide. The importance of the intermode dissipative and reactive couplings due to the neighboring waveguide is revealed and characterized from the peculiar Fano line shapes manifesting in transmission spectra. From the point of view of pure photonics, our study provides insight into a phenomenon that may have application to designing resonators with interesting nonlinear and quantum functionalities. From a broader perspective, it provides a simple model in which to study a fundamental feature of the theory of open systems, namely, the possibility of environment-mediated couplings—the off-diagonal photonic Lamb shift—between different modes of a system.

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APPENDIX: THE EIGENFREQUENCIES OF THE RESONATOR

As we mentioned in Sec. II, the frequency shift of the modes and their radiative decay rate as a function of the waveguide position below the disk resonator shown in Fig. 1 were obtained by solving an eigenvalue problem with suitable absorbing boundary conditions. In particular, to isolate the frequency shift and the decay rate of each mode taken independently of the others, in these first simulations we restricted our attention to a frequency region where the two radial family modes are well detuned from each other.

An example of the distortions that one could otherwise find is illustrated in Fig. 7. In terms of the analytical model of Sec. III, the complex eigenfrequencies obtained from the FEM calculation correspond to the eigenvalues of the full linear problem in (1). While in the general case these eigenfrequencies are strongly affected by the intermode coupling terms that are the main subject of this work, they recover the independent mode frequencies $\omega_j^o + \Delta_{jj} + i\Gamma_{jj}^{rad}/2$ when the modes are well separated in frequency.



FIG. 7. (Color online) Ratio of the decay rates and frequency shift of the resonator modes as obtained from finite-element simulations. In the left (a), (b) panels [the same plots as in Fig. 1(a)], the different radial family modes are well detuned from each other and effectively independent. In the right (c),(d) panels, the considered modes are mixed by the off-diagonal reactive and dissipative coupling terms.

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